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$N = 2$ sigma models for Ramond-Ramond backgrounds

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Abstract: Using the U(4) hybrid formalism, manifestly $N = (2, 2)$ worldsheet supersymmetric sigma models are constructed for the type-IIB superstring in Ramond-Ramond backgrounds. The Kahler potential in these $N = 2$ sigma models depends on four chiral and antichiral bosonic superfields and two chiral and antichiral fermionic superfields. When the Kahler potential is quadratic, the model is a free conformal field theory which describes a flat ten-dimensional target space with Ramond-Ramond flux and non-constant dilaton. For more general Kahler potentials, the model describes curved target spaces with Ramond-Ramond flux that are not plane-wave backgrounds. Ricci-flatness of the Kahler metric implies the on-shell conditions for the background up to the usual four-loop conformal anomaly.

Keywords: Superstrings and Heterotic Strings, Penrose limit and pp-wave background.

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Contents

1. Introduction

Because of the AdS/CFT correspondence [1], understanding the superstring in backgrounds with Ramond-Ramond (RR) flux is an important problem. For plane-wave RR backgrounds, one can use the light-cone Green-Schwarz (GS) formalism to compute the physical spectrum [2, 3]. In principle, the light-cone GS formalism can also be used to compute scattering amplitudes, however, there are complications caused by interaction-point operators and contact terms. For RR backgrounds which do not allow a light-cone gauge choice, one can use the classical covariant GS formalism to study supergravity properties of the background [4]. However, it is difficult to compute string-related properties of the background because of quantization problems in the covariant GS formalism.

An alternative approach to study RR backgrounds is to use the new covariant formalism involving pure spinors [5]. This formalism manifestly preserves all isometries of the background and is easy to quantize using a BRST operator. Unfortunately, although it is straightforward to construct quantizable actions for the superstring in $AdS_5 \times S^5$ [6] and plane-wave RR backgrounds [7] using this formalism, the resulting actions are non-linear and it is not yet known how to simplify or solve them.

Recently [8], it has been realized that another approach called the U(4) hybrid formalism [9] may be very useful for studying type-IIB RR backgrounds. This U(4) hybrid formalism is manifestly $N = (2, 2)$ worldsheet supersymmetric and contains four chiral and antichiral bosonic superfields, X^l and $X^{\bar{l}}$ for $l = 1$ to 4, two chiral and antichiral fermionic superfields, (Θ_L, Θ_R) and $(\overline{\Theta}_L, \overline{\Theta}_R)$, and two semi-chiral and semi-antichiral fermionic superfields, (W_L, W_R) and $(\overline{W}_L, \overline{W}_R)$. Although this formalism only manifestly preserves 25 of the 45 SO(9, 1) Lorentz transformations, it is also manifestly invariant under 20 of the 32 type-IIB supersymmetries. As was shown in [10], the formalism is a critical $N = 2$ superconformal field theory which is related by a field redefinition to the N=1 $\to N = 2$ embedding of the RNS superstring. And in light-cone gauge, where the fermionic superfields are gauged away, the U(4) hybrid formalism reduces to the light-cone GS formalism including the correct interaction-point operators [9].

In [8], the U(4) hybrid formalism was used to describe plane-wave RR backgrounds [11] in which all non-zero RR field strengths have a spacetime + index. (Throughout this paper, the $\pm$ index will always refer to the $x^{\pm} = (1/\sqrt{2})(x^9 \pm x^0)$ spacetime directions.) For plane-wave RR backgrounds, the U(4) worldsheet action depends trivially on the $(W, \overline{W})$ superfields and closely resembles the light-cone GS action [11]. However, unlike the light-cone GS action, the U(4) worldsheet action does not require contact terms and quantum superconformal invariance of the action implies the on-shell conditions of the background.[1]

Although plane-wave RR backgrounds are interesting to study because they are Penrose limits of the $AdS_5 \times S^5$ background, it would be useful to also have simple conformal field theory descriptions of more general RR backgrounds. As will be shown here, the U(4) hybrid formalism can not only be used to describe backgrounds with RR field strengths containing a spacetime + index, but can also be used to describe certain RR field strengths containing a spacetime $-$ index. Such backgrounds do not allow a light-cone gauge choice and therefore cannot be described using the light-cone GS formalism.

In the U(4) hybrid formalism, there are two special RR field strengths containing a spacetime $-$ index whose vertex operators are $\int d^2z \int d^4\kappa\, W_L \overline{W}_R$ and $\int d^2z \int d^4\kappa\, W_R \overline{W}_L$ where $(W_R, W_L, \overline{W}_R, \overline{W}_L)$ are the semi-chiral and semi-antichiral superfields. If these RR field strengths have non-zero flux, the W superfields satisfy auxiliary equations of motion and can be integrated out of the worldsheet action. The resulting worldsheet action is an $N = (2,2)$ sigma model which depends on four chiral and antichiral bosonic superfields, X^l and $\overline{X}^{\bar{l}}$, and two chiral and antichiral fermionic superfields, (Θ_L, Θ_R) and $(\overline{\Theta}_L, \overline{\Theta}_R)$, through a Kahler potential $S = \int d^2z \int d^4\kappa\, K(X, \overline{X}, \Theta, \overline{\Theta})$. The background is on-shell up to the usual four-loop conformal anomaly [14] when the Kahler metric $G_{M\overline{N}} = \partial_M \overline{\partial}_{\overline{N}} K$ is Ricci-flat [12].

It is interesting to ask what types of superstring backgrounds are described by the Kahler potential $K(X, \overline{X}, \Theta, \overline{\Theta})$. When the Kahler potential is quadratic, i.e. $K = X^l \overline{X}^{\bar{l}} + \Theta_L \overline{\Theta}_R + \Theta_R \overline{\Theta}_L$, it will be argued that the background is a flat ten-dimensional spacetime containing non-zero RR field strengths with a spacetime $-$ index and a non-constant dilaton field ϕ which depends quadratically on x^-. So this RR background with a non-constant dilaton is described by a free conformal field theory. It would be interesting to construct vertex operators and compute scattering amplitudes in this RR background using the free-field OPE's implied by the action.

More general Kahler potentials will be shown to describe certain curved backgrounds which contain non-zero RR field strengths with both spacetime + and $-$ indices. It should

[1] It was recently claimed that any plane-wave RR background which satisfies the supergravity equations at lowest order in α' is a consistent superstring background [18]. In the U(4) formalism, this claim implies that any plane-wave RR background which is superconformally invariant at one-loop is superconformally invariant at all loops. This is reasonable since, as was discussed in [8], plane-wave RR backgrounds involve interaction vertices which are quadratic in Θ. Since there are only four independent Θ's, any divergent counterterm can involve at most two interaction vertices. But above one-loop, all known counterterms in $N = 2$ sigma models involve more than two vertices. For example, the well-known four-loop R^4 counterterm [14] involves a minimum of four vertices. So if one could prove that all possible counterterms above one-loop in $N = 2$ sigma models involve more than two vertices, one will have proven the claim of [18].

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be possible to use standard $N = (2,2)$ superconformal methods to study the physical spectrum and scattering amplitudes in these RR backgrounds which do not allow a light-cone gauge choice. Hopefully, this information will be useful for learning more about the AdS/CFT correspondence.

In section 2 of this paper, the U(4) hybrid formalism and its field redefinition to the RNS formalism will be reviewed in a flat background. In section 3, the RR vertex operators $\int d^2zd^4\kappa\ W_L\overline{W}_R$ and $\int d^2zd^4\kappa\ W_R\overline{W}_L$ will be added to the flat action and the resulting linear $N = 2$ sigma model will be discussed. And in section 4, the properties of more general non-linear $N = 2$ sigma models using the U(4) hybrid formalism will be described.

2. U(4) hybrid formalism in a flat background

One way of understanding the U(4) hybrid formalism in a flat background is as a field redefinition from the N=1→N=2 embedding of the RNS formalism [10]. Recall that the critical N=1 description of the RNS superstring can be embedded into a critical $N = 2$ description by defining the $\hat{c} = 2$ $N = 2$ superconformal generators as

$$\begin{aligned} T &= T^{\text{matter}}_{N=1} + T^{\text{ghost}}_{N=1} + \frac{1}{2}\partial(bc + \xi\eta)\,, \\ G &= j_{BRST}\,, \\ \overline{G} &= b\,, \\ J &= bc + \xi\eta\,, \end{aligned} \tag{2.1}$$

where $Q = \int dz\ j_{BRST}$ is the RNS BRST operator and (ξ, η) come from bosonizing the N=1 super-reparameterization ghosts as $\beta = \partial\xi e^{-\phi}$ and $\gamma = \eta e^{\phi}$. Note that the term $(1/2)(bc + \xi\eta)$ in T shifts the conformal weights of the RNS ghosts so that G and $\overline{G}$ have conformal weight $\frac{3}{2}$. As discussed in [10], physical states in the RNS formalism can be defined as $N = 2$ superconformally invariant states with respect to these $N = 2$ generators, and RNS scattering amplitudes can be computed using standard $N = 2$ rules.

Although the RNS worldsheet variables transform in a complicated non-linear manner under the $N = 2$ superconformal transformations generated by $[T, G, \overline{G}, J]$, one can define a field redefinition to U(4) hybrid variables which transform linearly under these $N = 2$ transformations. These left-moving U(4) hybrid variables are [9]

$$[x^l, \overline{x}^{\bar{l}}, s^l_L, \overline{s}^{\bar{l}}_L, p_L, \overline{p}_L, \theta_L, \overline{\theta}_L, \lambda_L, \overline{\lambda}_L, w_L, \overline{w}_L] \quad \text{for} \quad l = 1 \quad \text{to} \quad 4 \tag{2.2}$$

and are related to the left-moving RNS variables $[x^\mu_{RNS}, \psi^\mu, b, c, \xi, \eta, \phi]$ for $\mu = 0$ to 9 by the field redefinition

$$\begin{aligned} x^l &= \frac{1}{\sqrt{2}}\left(x^l_{RNS} + ix^{l+4}_{RNS}\right), \\ \overline{x}^{\bar{l}} &= \frac{1}{\sqrt{2}}\left(x^l_{RNS} - ix^{l+4}_{RNS} + c\xi e^{-\phi}\left(\psi^l - i\psi^{l+4}\right)\right), \\ s^l &= \frac{1}{\sqrt{2}}\eta e^{\phi}\left(\psi^l + i\psi^{l+4}\right) + \frac{1}{\sqrt{2}}c\partial_L\left(x^l_{RNS} + ix^{l+4}_{RNS}\right), \end{aligned}$$

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$$
\begin{aligned}
\overline{s}^{\overline{l}} &= \frac{1}{\sqrt{2}}\xi e^{-\phi}\left(\psi^{l} - i\psi^{l+4}\right),\\
p_L &= b\eta e^{(3/2)\phi}\Sigma^{-++++} + e^{(1/2)\phi}\left(\partial_L x^-_{RNS}\Sigma^{+++++} + \frac{1}{\sqrt{2}}\partial_L\left(x^l_{RNS} + i x^{l+4}_{RNS}\right)\Sigma^{-\overline{l}}\right) -\\
&\quad - \partial_L\left(x^-_L e^{(1/2)\phi}\Sigma^{+++++}\right),\\
\overline{p}_L &= e^{-(1/2)\phi}\Sigma^{-----} - \partial_L\left(c x^-_L \xi e^{-(3/2)\phi}\Sigma^{+----}\right),\\
\theta_L &= e^{(1/2)\phi}\Sigma^{+++++},\\
\overline{\theta}_L &= c\xi e^{-(3/2)\phi}\Sigma^{+----},\\
\lambda_L &= \eta e^{(3/2)\phi}\left(\partial_L x^+_{RNS}\Sigma^{-++++} + \frac{1}{\sqrt{2}}\partial_L\left(x^l_{RNS} + i x^{l+4}_{RNS}\right)\Sigma^{+\overline{l}}\right) +\\
&\quad + c\partial_L\left(e^{(1/2)\phi}\Sigma^{+++++}\right) + b\eta\partial_L\eta e^{(5/2)\phi}\Sigma^{+++++},\\
\overline{\lambda}_L &= \xi e^{-(3/2)\phi}\Sigma^{+-----},\\
w_L &= Q\left(c\xi e^{-(1/2)\phi}\Sigma^{-++++} + x^-_L e^{(1/2)\phi}\Sigma^{+++++}\right),\\
\overline{w}_L &= x^-_L \xi e^{-(3/2)\phi}\Sigma^{+----},
\end{aligned}
\tag{2.3}
$$

where x^-_L and x^-_R are the right and left-moving contributions to

$$
x^-_{RNS} = \frac{1}{\sqrt{2}}(x^9_{RNS} - x^0_{RNS}) = x^-_L + x^-_R\,, \tag{2.4}
$$

$\Sigma^{\pm\pm\pm\pm\pm}$ are the 32 RNS spin fields constructed by bosonizing ψ^μ, and

$$
\begin{aligned}
\Sigma^{+\overline{l}} &= (\Sigma^{+-+++}, \Sigma^{++-++}, \Sigma^{+++-+}, \Sigma^{++++-})\,,\\
\Sigma^{-\overline{l}} &= (\Sigma^{--+++}, \Sigma^{-+-++}, \Sigma^{-++-+}, \Sigma^{-+++-})\,.
\end{aligned}
\tag{2.5}
$$

Although the field redefinition of (2.3) looks complicated, it can be derived by first defining x^l and $\overline{x}^{\overline{l}}$ such that $b_0(x^l) = Q(\overline{x}^{\overline{l}}) = 0$, which implies that x^l and $\overline{x}^{\overline{l}}$ are $N=2$ chiral and antichiral with respect to the generators of (2.1). One then defines $s^l = Q(x^l)$ and $\overline{s}^{\overline{l}} = b_0(\overline{x}^{\overline{l}})$. The next step is to define p_L and $\overline{p}_L$ such that $\int dz_L\ p_L$ and $\int dz_L\ \overline{p}_L$ are the $q^{-++++}_{1/2}$ and $q^{-----}_{-1/2}$ supersymmetry generators in the $+1/2$ and $-1/2$ picture of Friedan-Martinec-Shenker [15], and then adds total derivative terms to p_L and $\overline{p}_L$ so that they have non-singular OPE's with each other. θ_L and $\overline{\theta}_L$ are then defined to be conjugates to $\overline{p}_L$ and p_L. Finally, one defines $\lambda_L = Q(\theta_L)$ and $\overline{\lambda}_L = b_0(\overline{\theta}_L)$, and defines w_L and $\overline{w}_L$ to be conjugates to $\overline{\lambda}_L$ and λ_L which have non-singular OPE's with the other fields. Similarly, the field redefinition for the right-moving U(4) hybrid variables is obtained by replacing all left-moving fields with right-moving fields in (2.3).

In terms of the U(4) hybrid variables, one can check that the left-moving $N=2$ generators of (2.1) are mapped to

$$
\begin{aligned}
T_L &= \partial_L x^l \partial_L \overline{x}^{\overline{l}} - \frac{1}{2}(s^l_L\partial_L\overline{s}^{\overline{l}}_L + \overline{s}^{\overline{l}}_L\partial_L s^l_L) - \overline{p}_L\partial_L\theta_L - p_L\partial_L\overline{\theta}_L +\\
&\quad + \frac{1}{2}(w_L\partial_L\overline{\lambda}_L - \overline{\lambda}_L\partial_L w_L) + \frac{1}{2}(\overline{w}_L\partial_L\lambda_L - \lambda_L\partial_L\overline{w}_L)\,,
\end{aligned}
$$

$$
\begin{aligned}
G_L &= s^l_L \partial_L \overline{x}^{\overline{l}} + \lambda_L \overline{p}_L + w_L \partial_L \overline{\theta}_L\,, \\
\overline{G}_L &= \overline{s}^{\overline{l}}_L \partial_L x^l + \overline{\lambda}_L p_L + \overline{w}_L \partial_L \theta_L\,, \\
J_L &= s^l_L \overline{s}^{\overline{l}}_L - w_L \overline{\lambda}_L + \overline{w}_L \lambda_L\,,
\end{aligned} \tag{2.6}
$$

and the RNS worldsheet action is mapped to

$$
\begin{aligned}
S = \int d^2 z \Big[&\partial_L x^l \partial_R x^{\overline{l}} - s^l_L \partial_R \overline{s}^{\overline{l}}_L - s^l_R \partial_L \overline{s}^{\overline{l}}_R - p_L \partial_R \overline{\theta}_L - \overline{p}_L \partial_R \theta_L + w_L \partial_R \overline{\lambda}_L + \\
&+ \overline{w}_L \partial_R \lambda_L - p_R \partial_L \overline{\theta}_R - \overline{p}_R \partial_L \theta_R + w_R \partial_L \overline{\lambda}_R + \overline{w}_R \partial_L \lambda_R \Big]\,.
\end{aligned} \tag{2.7}
$$

So the U(4) variables of (2.2) satisfy free-field OPE's and transform linearly under the $N = 2$ superconformal transformations generated by (2.6).

To construct these linearly transforming variables, it was necessary to explicitly separate x^-_{RNS} into its left and right-moving parts as $x^-_{RNS} = x^-_L + x^-_R$. As will now be discussed, this separation implies that physical states in the U(4) hybrid formalism must not only be $N = 2$ superconformally invariant, but must also satisfy an additional global constraint.

Using the field redefinition of (2.3), one can check that

$$
\lambda_L \overline{\lambda}_L - \overline{\theta}_L \partial_L \theta_L = \partial_L x^+_{RNS} \qquad \text{and} \qquad \lambda_R \overline{\lambda}_R - \overline{\theta}_R \partial_R \theta_R = \partial_R x^+_{RNS}\,. \tag{2.8}
$$

Since $\oint dz_L\ \partial_L x^+_{RNS} = \oint dz_R\ \partial_R x^+_{RNS}$ for any closed contour, the U(4) hybrid variables satisfy the global constraint

$$
\oint dz_L (\lambda_L \overline{\lambda}_L - \overline{\theta}_L \partial_L \theta_L) - \oint dz_R (\lambda_R \overline{\lambda}_R - \overline{\theta}_R \partial_R \theta_R) = 0\,. \tag{2.9}
$$

Using canonical commutation relations, the constraint of (2.9) generates the global isometry

$$
\begin{aligned}
\delta w_L &= \alpha \lambda_L\,, & \delta \overline{w}_L &= \alpha \overline{\lambda}_L\,, & \delta w_R &= -\alpha \lambda_R\,, & \delta \overline{w}_R &= -\alpha \overline{\lambda}_R\,, \\
\delta p_L &= -\alpha \partial_L \theta_L\,, & \delta \overline{p}_L &= -\alpha \partial_L \overline{\theta}_L\,, & \delta p_R &= \alpha \partial_R \theta_R\,, & \delta \overline{p}_R &= \alpha \partial_R \overline{\theta}_R\,.
\end{aligned} \tag{2.10}
$$

And from (2.3), one can check that this isometry corresponds in the RNS formalism to

$$
\delta x^-_L = \alpha\,, \qquad \delta x^-_R = -\alpha\,. \tag{2.11}
$$

So physical states in the U(4) hybrid formalism must satisfy the global constraint of (2.9), which implies that they only depend on x^-_L and x^-_R in the combination $x^-_{RNS} = x^-_L + x^-_R$.

Since the U(4) hybrid variables transform linearly under $N = (2,2)$ worldsheet supersymmetry, it is convenient to combine them into the $N = (2,2)$ chiral and antichiral superfields

$$
\begin{aligned}
X^l(\kappa_L, \kappa_R) &= x^l + \kappa_L s^l_L + \kappa_R s^l_R + \kappa_L \kappa_R t^l\,, & \overline{X}^{\overline{l}}(\overline{\kappa}_L, \overline{\kappa}_R) &= \overline{x}^{\overline{l}} + \overline{\kappa}_L \overline{s}^{\overline{l}}_L + \overline{\kappa}_R \overline{s}^{\overline{l}}_R + \overline{\kappa}_L \overline{\kappa}_R \overline{t}^{\overline{l}}\,, \\
\Theta_L(\kappa_L, \kappa_R) &= \theta_L + \kappa_L \lambda_L + \cdots\,, & \Theta_R(\kappa_L, \kappa_R) &= \theta_R + \kappa_R \lambda_R + \cdots\,, \\
\overline{\Theta}_L(\overline{\kappa}_L, \overline{\kappa}_R) &= \overline{\theta}_L + \overline{\kappa}_L \overline{\lambda}_L + \cdots\,, & \overline{\Theta}_R(\overline{\kappa}_L, \overline{\kappa}_R) &= \overline{\theta}_R + \overline{\kappa}_R \overline{\lambda}_R + \cdots\,,
\end{aligned} \tag{2.12}
$$

and the $N=(2,2)$ semi-chiral and semi-antichiral superfields,

$$\begin{aligned}
W_L(\kappa_L,\kappa_R,\overline{\kappa}_L) &= \cdots + \kappa_L w_L - \kappa_L\overline{\kappa}_L p_L + \cdots\,,\\
W_R(\kappa_L,\kappa_R,\overline{\kappa}_R) &= \cdots + \kappa_R w_R - \kappa_R\overline{\kappa}_R p_R + \cdots\,,\\
\overline{W}_L(\kappa_L,\overline{\kappa}_L,\overline{\kappa}_R) &= \cdots + \overline{\kappa}_L \overline{w}_L - \overline{\kappa}_L\kappa_L \overline{p}_L + \cdots\,,\\
\overline{W}_R(\kappa_R,\overline{\kappa}_L,\overline{\kappa}_R) &= \cdots + \overline{\kappa}_R \overline{w}_R - \overline{\kappa}_R\kappa_R \overline{p}_R + \cdots\,,
\end{aligned} \tag{2.13}$$

where t^l, $\overline{t}^{\overline{l}}$ and $\cdots$ denote auxiliary fields which can be gauged away or vanish on-shell in a flat background. Defining

$$\begin{aligned}
D_L &= \frac{\partial}{\partial\kappa_L} + \frac{1}{2}\overline{\kappa}_L\partial_L\,, \qquad & D_R &= \frac{\partial}{\partial\kappa_R} + \frac{1}{2}\overline{\kappa}_R\partial_R\,,\\
\overline{D}_L &= \frac{\partial}{\partial\overline{\kappa}_L} + \frac{1}{2}\kappa_L\partial_L\,, \qquad & \overline{D}_R &= \frac{\partial}{\partial\overline{\kappa}_R} + \frac{1}{2}\kappa_R\partial_R\,,
\end{aligned} \tag{2.14}$$

these superfields are constrained to satisfy the chirality constraints

$$\begin{aligned}
\overline{D}_L X^l &= \overline{D}_R X^l = D_L\overline{X}^{\overline{l}} = D_R\overline{X}^{\overline{l}} = 0\,,\\
\overline{D}_L\Theta_L &= \overline{D}_R\Theta_L = \overline{D}_L\Theta_R = \overline{D}_R\Theta_R = D_L\overline{\Theta}_L = D_R\overline{\Theta}_L = D_L\overline{\Theta}_R = D_R\overline{\Theta}_R = 0\,,\\
\overline{D}_R W_L &= \overline{D}_L W_R = D_R\overline{W}_L = D_L\overline{W}_R = 0\,.
\end{aligned} \tag{2.15}$$

In terms of these $N=(2,2)$ superfields, the $N=2$ left-moving stress-tensor of (2.6) is

$$\mathcal{T}_L = (\overline{D}_L\overline{W}_L)D_L\Theta_L - (D_L W_L)\overline{D}_L\overline{\Theta}_L + D_L X^l \overline{D}_L\overline{X}^{\overline{l}}\,,$$

the worldsheet action of (2.7) is

$$S = \int d^6 Z\left[X^l\overline{X}^{\overline{l}} + W_L\overline{\Theta}_L + W_R\overline{\Theta}_R + \overline{W}_L\Theta_L + \overline{W}_R\Theta_R\right], \tag{2.16}$$

the global constraint of (2.9) is

$$\int d^3 Z_L\ \Theta_L\overline{\Theta}_L - \int d^3 Z_R\ \Theta_R\overline{\Theta}_R = 0\,, \tag{2.17}$$

and the isometry of (2.17) is

$$\delta W_L = \alpha\Theta_L\,, \quad \delta W_R = -\alpha\Theta_R\,, \quad \delta\overline{W}_L = \alpha\overline{\Theta}_L\,, \quad \delta\overline{W}_R = -\alpha\overline{\Theta}_R\,, \tag{2.18}$$

where $\int d^6 Z$ denotes $\int d^2 z\overline{D}_L\overline{D}_R D_L D_R$, $\int d^3 Z_L$ denotes $(1/2)\oint dz_L(\overline{D}_L D_L - D_L\overline{D}_L)$ and $\int d^3 Z_R$ denotes $(1/2)\oint dz_R(\overline{D}_R D_R - D_R\overline{D}_R)$.

With respect to SO(9,1) super-Poincaré transformations, the U(4) hybrid formalism is manifestly invariant under 20 of the 32 type-IIB spacetime supersymmetries which are generated by

$$\begin{aligned}
q_L^{+l} &= \int d^3 Z_L\ X^l\overline{\Theta}_L\,, \qquad & q_L^{+\overline{l}} &= \int d^3 Z_L\ \overline{X}^{\overline{l}}\Theta_L\,,\\
q_L^{-----} &= \int d^3 Z_L\ W_L\,, \qquad & q_L^{-++++} &= \int d^3 Z_L\ \overline{W}_L\,,\\
q_R^{+l} &= \int d^3 Z_R\ X^l\overline{\Theta}_R\,, \qquad & q_R^{+\overline{l}} &= \int d^3 Z_R\ \overline{X}^{\overline{l}}\Theta_R\,,\\
q_R^{-----} &= \int d^3 Z_R\ W_R\,, \qquad & q_R^{-++++} &= \int d^3 Z_R\ \overline{W}_R\,,
\end{aligned} \tag{2.19}$$

under 25 of the 45 Lorentz transformations generated by

$$
\begin{aligned}
M^{l\overline{m}} &= \int d^3 Z_L\ X^l \overline{X}^{\overline{m}} + \int d^3 Z_R\ X^l \overline{X}^{\overline{m}}\,,\\
M^{+l} &= \int d^3 Z_L\ X^l \Theta_L \overline{\Theta}_L + \int d^3 Z_R\ X^l \Theta_R \overline{\Theta}_R\,,\\
M^{+\overline{l}} &= \int d^3 Z_L\ \overline{X}^{\overline{l}} \Theta_L \overline{\Theta}_L + \int d^3 Z_R\ \overline{X}^{\overline{l}} \Theta_R \overline{\Theta}_R\,,\\
M^{+-} &= \int d^3 Z_L\ (W_L \overline{\Theta}_L - \overline{W}_L \Theta_L) + \int d^3 Z_R\ (W_R \overline{\Theta}_R - \overline{W}_R \Theta_R)\,,
\end{aligned}
\tag{2.20}
$$

and under 9 of the 10 translations generated by

$$
\begin{aligned}
P^l &= -\int d^3 Z_L\ X^l = -\int d^3 Z_R\ X^l\,,\\
P^{\overline{l}} &= \int d^3 Z_L\ \overline{X}^{\overline{l}} = \int d^3 Z_R\ \overline{X}^{\overline{l}}\,,\\
P^+ &= \int d^3 Z_L\ \Theta_L \overline{\Theta}_L = \int d^3 Z_R\ \Theta_R \overline{\Theta}_R\,.
\end{aligned}
\tag{2.21}
$$

Note that the translation generator P^- does not act on the U(4) hybrid variables since the field redefinition of (2.3) does not involve the zero mode of x^+_{RNS}.

In this paper, only the type-IIB version of the U(4) formalism will be discussed. As was recently shown in [19], treating the type-IIA superstring in an $N=(2,2)$ worldsheet supersymmetric manner requires switching one of the four superfields X^l from a chiral superfield to a twisted-chiral superfield which breaks the manifest U(4) down to U(1) $\times$ U(3). For the heterotic version of the U(4) formalism, only $N=(2,0)$ worldsheet supersymmetry is present and the right-moving sector of the superstring is the same as in the RNS formalism.

3. Linear $N=2$ sigma model

In a recent paper with Maldacena [8], it was shown that the U(4) hybrid formalism can be generalized to plane-wave backgrounds in which the non-zero RR field strengths carry a spacetime + index. These RR field strengths appear in the worldsheet action through the vertex operator

$$
\int d^6 Z [f_1(X,\overline{X}) \Theta_L \Theta_R + f_2(X,\overline{X}) \overline{\Theta}_L \Theta_R - \overline{f}_1(X,\overline{X}) \overline{\Theta}_L \overline{\Theta}_R - \overline{f}_2(X,\overline{X}) \Theta_L \overline{\Theta}_R]\,,
\tag{3.1}
$$

which is constructed from left-right products of the spacetime supersymmetry generators $(q_L^{+l}, q_L^{+\overline{l}})$ and $(q_R^{+l}, q_R^{+\overline{l}})$ of (2.19). In this section, the U(4) hybrid formalism will be generalized to a background containing certain RR field strengths containing a spacetime $-$ index. These field strengths will couple through the vertex operator

$$
\int d^6 Z [f W_L \overline{W}_R - \overline{f} \overline{W}_L W_R]\,,
\tag{3.2}
$$

which is constructed from left-right products of the spacetime supersymmetry generators $(q_L^{-----}, q_L^{-++++})$ and $(q_R^{-----}, q_R^{-++++})$ of (2.19). Although f and $\overline{f}$ will be assumed to be constants in this paper, it might be possible to consider more general RR vertex operators which depend on both W and X.

If one defines

$$F^{\alpha\beta} = F^{\mu}_{(1)}\gamma^{\alpha\beta}_{\mu} + \frac{1}{6}F^{\mu\nu\rho}_{(3)}\gamma^{\alpha\beta}_{\mu\nu\rho} + \frac{1}{480}F^{\mu\nu\rho\sigma\tau}_{(5)}\gamma^{\alpha\beta}_{\mu\nu\rho\sigma\tau}\,, \tag{3.3}$$

where $[F^{\mu}_{(1)}, F^{\mu\nu\rho}_{(3)}, F^{\mu\nu\rho\sigma\tau}_{(5)}]$ are the type-IIB RR field strengths and $(\alpha, \beta) = 1$ to 16 are Majorana-Weyl spinor indices, then turning on the vertex operator of (3.2) corresponds to giving $e^{\phi}F^{\alpha\beta}$ the background value

$$e^{\phi}F^{\alpha\beta} = \frac{1}{2}M^{\alpha}_{\gamma}(\gamma^{-})^{\gamma\delta}\overline{M}^{\beta}_{\delta}f + \frac{1}{2}\overline{M}^{\alpha}_{\gamma}(\gamma^{-})^{\gamma\delta}M^{\beta}_{\delta}\overline{f} \tag{3.4}$$

where ϕ is the dilaton and

$$\begin{aligned} M^{\beta}_{\alpha} &= \frac{1}{4}[(\gamma^1 + i\gamma^5)(\gamma^2 + i\gamma^6)(\gamma^3 + i\gamma^7)(\gamma^4 + i\gamma^8)]_{\alpha}{}^{\beta}\,, \\ \overline{M}^{\beta}_{\alpha} &= \frac{1}{4}[(\gamma^1 - i\gamma^5)(\gamma^2 - i\gamma^6)(\gamma^3 - i\gamma^7)(\gamma^4 - i\gamma^8)]_{\alpha}{}^{\beta} \end{aligned} \tag{3.5}$$

are matrices which select the appropriate bispinor components of the RR field strength. So adding the vertex operator of (3.2) corresponds to giving non-zero flux to the components $F^{(1)}_{-}$, $F^{(3)}_{-l\bar{l}}$, and $F^{(5)}_{-lm\bar{l}\bar{m}}$ with relative coefficients which depend on f and $\overline{f}$.

When f and $\overline{f}$ are constants, the vertex operator of (3.2) is $\int d^2z(fp_L\overline{p}_R + \overline{f}p_R\overline{p}_L)$. So the worldsheet action of (2.7) or (2.16) is still quadratic after adding this vertex operator. In the presence of this vertex operator, the equations of motion for W and $\overline{W}$ become auxiliary and one can integrate them out [13] to obtain the linear sigma model action

$$\begin{aligned} S &= \int d^6Z\;(X^l\overline{X}^{\bar{l}} + \overline{f}^{-1}\Theta_L\overline{\Theta}_R + f^{-1}\Theta_R\overline{\Theta}_L) \\ &= \int d^2z\Big[\partial_L x^l\partial_R\overline{x}^{\bar{l}} + \overline{f}^{-1}(\partial_R\theta_L)(\partial_L\overline{\theta}_R) + f^{-1}(\partial_L\theta_R)(\partial_R\overline{\theta}_L) - \\ &\qquad - s^l_L\partial_R\overline{s}^{\bar{l}}_L - s^l_R\partial_L\overline{s}^{\bar{l}}_R + w_L\partial_R\overline{\lambda}_L + \overline{w}_L\partial_R\lambda_L + w_R\partial_L\overline{\lambda}_R + \overline{w}_R\partial_L\lambda_R\Big]\,. \end{aligned} \tag{3.6}$$

Note that $(\Theta_L, \overline{\Theta}_L)$ and $(\Theta_R, \overline{\Theta}_R)$ are no longer left and right-moving functions on-shell. In components,

$$\begin{aligned} \Theta_L(\kappa_L, \kappa_R) &= \theta_L + \kappa_L\lambda_L + \overline{f}\kappa_R w_R + \cdots, & \Theta_R(\kappa_L, \kappa_R) &= \theta_R + \kappa_R\lambda_R + f\kappa_L w_L + \cdots, \\ \overline{\Theta}_L(\overline{\kappa}_L, \overline{\kappa}_R) &= \overline{\theta}_L + \overline{\kappa}_L\overline{\lambda}_L - f\overline{\kappa}_R\overline{w}_R + \cdots, & \overline{\Theta}_R(\overline{\kappa}_L, \overline{\kappa}_R) &= \overline{\theta}_R + \overline{\kappa}_R\overline{\lambda}_R - \overline{f}\overline{\kappa}_L\overline{w}_L + \cdots, \end{aligned} \tag{3.7}$$

where $\cdots$ are auxiliary fields which vanish on-shell. The left and right-moving stress-tensors are now

$$\begin{aligned} \mathcal{T}_L &= -\overline{f}^{-1}(\overline{D}_L\overline{\Theta}_R)(D_L\Theta_L) - f^{-1}(D_L\Theta_R)(\overline{D}_L\overline{\Theta}_L) + D_LX^l\overline{D}_L\overline{X}^{\bar{l}}\,, \\ \mathcal{T}_R &= -f^{-1}(\overline{D}_R\overline{\Theta}_L)(D_R\Theta_R) - \overline{f}^{-1}(D_R\Theta_L)(\overline{D}_R\overline{\Theta}_R) + D_RX^l\overline{D}_R\overline{X}^{\bar{l}}\,, \end{aligned} \tag{3.8}$$

and are still quadratic in this RR background.

Unlike the U(4) formalism in the plane-wave RR backgrounds discussed in [8], the global constraint of (2.17) needs to be modified in this RR background since $(\Theta_L, \overline{\Theta}_L)$ are no longer left-moving and $(\Theta_R, \overline{\Theta}_R)$ are no longer right-moving. To find the correct modification to (2.17), note that the action of (3.6) is invariant under the global transformation

$$\delta\Theta_L = -\alpha\overline{f}\Theta_R\,, \quad \delta\Theta_R = \alpha f\Theta_L\,, \quad \delta\overline{\Theta}_L = \alpha f\overline{\Theta}_R\,, \quad \delta\overline{\Theta}_R = -\alpha\overline{f}\overline{\Theta}_L\,. \tag{3.9}$$

Furthermore, the auxiliary equations of motion for W and $\overline{W}$ imply that they transform under (3.9) in the same way as in (2.18).

Using the Noether method, the invariance under (3.9) implies that

$$J_L = \overline{D}_L D_L(\Theta_L\overline{\Theta}_L - \Theta_R\overline{\Theta}_R)\,, \quad J_R = \overline{D}_R D_R(\Theta_L\overline{\Theta}_L - \Theta_R\overline{\Theta}_R)\,, \tag{3.10}$$

is a conserved current satisfying $\partial_R J_L + \partial_L J_R = 0$. One can therefore restrict physical states to carry zero charge with respect to this current, i.e. to satisfy the global constraint

$$\int d^3 Z_L(\Theta_L\overline{\Theta}_L - \Theta_R\overline{\Theta}_R) + \int d^3 Z_R(\Theta_L\overline{\Theta}_L - \Theta_R\overline{\Theta}_R) = 0\,. \tag{3.11}$$

Or in components,

$$\begin{aligned}\oint dz_L(\lambda_L\overline{\lambda}_L + f\overline{f}w_L\overline{w}_L - \overline{\theta}_L\partial_L\theta_L + \overline{\theta}_R\partial_L\theta_R) = \\ = \oint dz_R(\lambda_R\overline{\lambda}_R + f\overline{f}w_R\overline{w}_R - \overline{\theta}_R\partial_R\theta_R + \overline{\theta}_L\partial_R\theta_L)\,.\end{aligned} \tag{3.12}$$

Note that this global constraint reduces to (2.9) when $f = \overline{f} = 0$ and $(\theta_{L/R}, \overline{\theta}_{L/R})$ are left/right-moving.

Since the worldsheet action of (3.6) is $N = 2$ superconformal invariant at the quantum level, an obvious question is what on-shell supergravity background is it describing. From the form of the action, it is clear that the metric is flat and there is non-zero RR flux in the directions described in (3.4). In string gauge, the graviton equation of motion implies that

$$e^{-2\phi}(R_{\mu\nu} - 2\nabla_\mu\nabla_\nu\phi) + (F^2)_{\mu\nu} = 0\,. \tag{3.13}$$

Using the RR field strength of (3.4), the only non-zero component of $(F^2)_{\mu\nu}$ is $(F^2)_{--} = 4e^{-2\phi}f\overline{f}$. So the background satisfies (3.13) if $R_{\mu\nu} = 0$, $(F^2)_{--} = 4e^{-2\phi}f\overline{f}$, and

$$\phi(x) = f\overline{f}(x^-)^2 + \phi_0\,. \tag{3.14}$$

One can easily check that this choice of ϕ and F is a solution of all the supergravity equations of motion in string gauge. Note that $e^{-\phi} \to 0$ as $|x^-| \to \infty$, so the energy density of this classical supergravity solution is everywhere finite. Furthermore, one can argue that this solution is not affected by α' corrections to the supergravity equations because of the inability to construct Lorentz-invariant quantities out of ϕ, F, and their derivatives.

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So even though the worldsheet action of (3.6) is free, it describes a non-trivial supergravity background with RR flux and non-constant dilaton. It would be interesting to construct vertex operators and compute scattering amplitudes using this quadratic worldsheet action and compare with analogous supergravity computations.

Since the action of (3.6) is so simple, it seems reasonable to look for a generalization of the field redefinition of (2.3) in this RR background with non-constant ϕ. Although it is not known how to construct RNS sigma model actions in RR backgrounds, there exists an alternative hybrid formalism which was developed with Gukov and Vallilo in [16] for describing compactifications to two dimensions. When the eight-dimensional compactification manifold is flat, the worldsheet variables in this $d = 2$ hybrid formalism are almost the same as in the U(4) hybrid formalism and consist of

$$\left[x^+, x^-, x^l, \overline{x}^{\bar{l}}, s_L^l, s_R^l, \overline{s}_L^{\bar{l}}, \overline{s}_R^{\bar{l}}, \theta_L, \theta_R, \overline{\theta}_L, \overline{\theta}_R, p_L, p_R, \overline{p}_L, \overline{p}_R, \sigma_L, \sigma_R, \rho_L, \rho_R\right], \tag{3.15}$$

where (ρ_L, σ_L) and (ρ_R, σ_R) are left and right-moving chiral bosons. So the only difference between the worldsheet variables of (3.15) and the U(4) hybrid variables is that $[x^+, x^-, \rho_L, \sigma_L, \rho_R, \sigma_R]$ is exchanged with $[\lambda_L, \lambda_R, \overline{\lambda}_L, \overline{\lambda}_R, w_L, w_R, \overline{w}_L, \overline{w}_R]$.

It was shown in [16] how to construct a sigma model action for RR backgrounds using the $d = 2$ hybrid formalism, so it should be possible to find the field redefinition which maps the U(4) and $d = 2$ hybrid formalisms into each other in the background of (3.4). As in a flat background, this field redefinition will require splitting x^- into x_L^- and x_R^-, and there will be a resulting global constraint on the U(4) variables. One should be able to verify that this global constraint is (3.11) and that the isometry of (3.9) is generated by $\delta x_L^- = \alpha$ and $\delta x_R^- = -\alpha$ as in (2.11). Also, it should be possible to understand the non-constant dilaton of (3.14) as coming from a non-trivial jacobian in the field redefinition.

4. Non-linear $N = 2$ sigma model

The linear sigma model $S = \int d^6 Z\, (X^l \overline{X}^{\bar{l}} + \overline{f}^{-1} \Theta_L \overline{\Theta}_R + f^{-1} \Theta_R \overline{\Theta}_L)$ of the previous section 3 has an obvious generalization to the non-linear sigma model

$$S = \int d^6 Z\ K(X, \overline{X}, \Theta, \overline{\Theta}), \tag{4.1}$$

where $K(X, \overline{X}, \Theta, \overline{\Theta})$ is the Kahler potential.[2] In order to describe a consistent superstring background, this non-linear sigma model must be $N = (2, 2)$ superconformal invariant at the quantum level and must be invariant under a global isometry analogous to (3.9).

[2] One can also add to the non-linear sigma model of (4.1) the Fradkin-Tseytlin-like term $\alpha' \int d^2 z [\int d\kappa_L d\kappa_R\ \Phi(X, \Theta) R + \int d\overline{\kappa}_L d\overline{\kappa}_R\ \overline{\Phi}(\overline{X}, \overline{\Theta}) \overline{R}]$ where $\Phi(X, \Theta)$ and $\overline{\Phi}(\overline{X}, \overline{\Theta})$ are chiral and antichiral target-space superfields and R and $\overline{R}$ are chiral and antichiral worldsheet superfields which describe the $N = (2, 2)$ supercurvature. In components, $R = \cdots + \kappa_R \kappa_L (r + it)$ and $\overline{R} = \cdots + \overline{\kappa}_R \overline{\kappa}_L (r - it)$ where r is the worldsheet curvature and t is the U(1) field strength for worldsheet R-symmetry [17]. So this term contains the usual $\alpha' \int d^2 z\ \phi r$ coupling where $\phi = \Phi + \overline{\Phi}$ is the spacetime dilaton superfield.

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Although it would be very interesting to study the most general action which satisfies these conditions, we shall restrict our attention in this paper to the action

$$S = \int d^6 Z \Big[k(X,\overline{X}) + a(X,\overline{X})\Theta_L\Theta_R + \overline{a}(X,\overline{X})\overline{\Theta}_R\overline{\Theta}_L + \\ + b(X,\overline{X}) \left(\frac{\sqrt{f}}{\sqrt{\overline{f}}}\Theta_L\overline{\Theta}_R + \frac{\sqrt{\overline{f}}}{\sqrt{f}}\Theta_R\overline{\Theta}_L \right) + d(X,\overline{X})\Theta_L\overline{\Theta}_L\Theta_R\overline{\Theta}_R \Big] , \quad (4.2)$$

which is invariant under the isometry of (3.9),

$$\delta\Theta_L = -\alpha\overline{f}\Theta_R , \quad \delta\Theta_R = \alpha f\Theta_L , \quad \delta\overline{\Theta}_L = \alpha f\overline{\Theta}_R , \quad \delta\overline{\Theta}_R = -\alpha\overline{f}\overline{\Theta}_L . \quad (4.3)$$

So physical states in this background must be $N = 2$ superconformal invariant and must satisfy the global constraint

$$\int d^3 Z_L \sqrt{f\overline{f}} b(X,\overline{X})(\Theta_L\overline{\Theta}_L - \Theta_R\overline{\Theta}_R) + \int d^3 Z_R \sqrt{f\overline{f}} b(X,\overline{X})(\Theta_L\overline{\Theta}_L - \Theta_R\overline{\Theta}_R) = 0 , \quad (4.4)$$

which generates the isometry of (4.3). As will now be shown, the action of (4.2) describes a background which includes both RR fluxes with a spacetime + index that appear in plane-wave backgrounds and RR fluxes with a spacetime $-$ index that were discussed in the previous section 3.

To learn what superstring background is described by (4.2), it is useful to put back the $(W,\overline{W})$ dependence in the action. Defining the isometry transformation of W and $\overline{W}$ as in the previous sections, i.e.

$$\delta W_L = \alpha\Theta_L , \quad \delta W_R = -\alpha\Theta_R , \quad \delta\overline{W}_L = \alpha\overline{\Theta}_L , \quad \delta\overline{W}_R = -\alpha\overline{\Theta}_R , \quad (4.5)$$

the term

$$\int d^6 Z [W_L\overline{\Theta}_L + W_R\overline{\Theta}_R + \overline{W}_L\Theta_L + \overline{W}_R\Theta_R + fW_L\overline{W}_R + \overline{f}W_R\overline{W}_L] \quad (4.6)$$

is invariant under the isometry of (4.3) and (4.5). After adding (4.6) to the action of (4.2) and integrating out W and $\overline{W}$, the only effect is to shift $b(X,\overline{X}) \to b(X,\overline{X}) + \frac{1}{\sqrt{f\overline{f}}}$. So (4.2) is equivalent to

$$S = S_0 + \int d^6 Z \Big[\widehat{k}(X,\overline{X}) + a(X,\overline{X})\Theta_L\Theta_R + \overline{a}(X,\overline{X})\overline{\Theta}_R\overline{\Theta}_L + fW_L\overline{W}_R + \overline{f}W_R\overline{W}_L + \\ + \widehat{b}(X,\overline{X}) \left(\frac{\sqrt{f}}{\sqrt{\overline{f}}}\Theta_L\overline{\Theta}_R + \frac{\sqrt{\overline{f}}}{\sqrt{f}}\Theta_R\overline{\Theta}_L \right) + d(X,\overline{X})\Theta_L\overline{\Theta}_L\Theta_R\overline{\Theta}_R \Big] , \quad (4.7)$$

where S_0 is the action of (2.16) in a flat background,

$$\widehat{k}(X,\overline{X}) = k(X,\overline{X}) - X^l\overline{X}^{\overline{l}} \qquad \text{and} \qquad \widehat{b}(X,\overline{X}) = b(X,\overline{X}) - \frac{1}{\sqrt{f\overline{f}}} . \quad (4.8)$$

By comparing with the massless vertex operators in a flat background, one can easily determine the linearized values of the supergravity background fields which contribute to (4.7). One finds

$$
\begin{aligned}
g_{l\overline{m}} &= \delta_{l\overline{m}} + \partial_l \overline{\partial}_{\overline{m}} \widehat{k}\,, \qquad g_{++} = -d + (\overline{\partial}_{\overline{l}} a)(\partial_l \overline{a}), \quad g_{+-} = 1\,, \\
\phi(x) &= \phi_0 + \frac{1}{4}(x^-)^2 f \overline{f}\,, \\
e^\phi F^{\alpha\beta} &= \frac{1}{2}(\gamma^l)^{\alpha\gamma} \overline{M}^\delta_\gamma \gamma^+_{\delta\kappa} (\gamma^m)^{\kappa\beta} \nabla_l \nabla_m \overline{a} + \frac{1}{2}(\gamma^{\overline{l}})^{\alpha\gamma} M^\delta_\gamma \gamma^+_{\delta\kappa} (\gamma^{\overline{m}})^{\kappa\beta} \overline{\nabla}_{\overline{l}} \overline{\nabla}_{\overline{m}} a + \\
&\quad + \frac{1}{8}(\gamma^{\overline{l}})^{\alpha\gamma} M^\delta_\gamma \gamma^+_{\delta\kappa} \overline{M}^\kappa_\rho (\gamma^m)^{\rho\beta} \frac{\sqrt{f}}{\sqrt{\overline{f}}} \overline{\nabla}_{\overline{l}} \nabla_m \widehat{b} + \frac{1}{8}(\gamma^l)^{\alpha\gamma} \overline{M}^\delta_\gamma \gamma^+_{\delta\kappa} M^\kappa_\rho (\gamma^{\overline{m}})^{\rho\beta} \frac{\sqrt{\overline{f}}}{\sqrt{f}} \nabla_l \overline{\nabla}_{\overline{m}} \widehat{b} + \\
&\quad + \frac{1}{2} M^\alpha_\delta (\gamma^-)^{\delta\kappa} \overline{M}^\beta_\kappa f + \frac{1}{2} \overline{M}^\alpha_\delta (\gamma^-)^{\delta\kappa} M^\beta_\kappa \overline{f}\,,
\end{aligned} \tag{4.9}
$$

where $(g_{l\overline{m}}, g_{++}, g_{+-})$ are the non-zero components of the metric, $F^{\alpha\beta}$ are components of the RR field strength written in the bispinor notation of (3.3), $(\gamma^l)^{\alpha\beta} = e^l_c(\gamma^c)^{\alpha\beta}$ and $(\gamma^{\overline{l}})^{\alpha\beta} = e^{\overline{l}}_c(\gamma^c)^{\alpha\beta}$, $c = 0$ to 9 is a tangent-space vector index, $(e^l_c, e^{\overline{l}}_c, e^+_c, e^-_c)$ is the vielbein satisfying $e^l_c e^{\overline{m}}_d \eta^{cd} = g^{l\overline{m}}$ and $e^l_c e^m_d \eta^{cd} = e^{\overline{l}}_c e^{\overline{m}}_d \eta^{cd} = 0$, and M^β_α and $\overline{M}^\beta_\alpha$ are defined in (3.5).

Since the background values of (4.9) have been determined from infinitesimal vertex operators, they are only guaranteed to be correct to linearized order in the fields

$$[\widehat{k}, a, \overline{a}, \widehat{b}, d, f, \overline{f}]\,. \tag{4.10}$$

But certain backreactions which are quadratic in these fields can be determined from other analysis. For example, the quadratic term $(\overline{\partial}_{\overline{l}} a)(\partial_l \overline{a})$ in g_{++} comes from integrating out the auxiliary variables t^l and $\overline{t}^{\overline{l}}$ of (2.12) in a plane-wave background. And the quadratic dependence on f in $\phi(x)$ comes from the solution of (3.14) in a pure RR background. However, there are certainly other backreactions which are cubic or higher-order in the fields of (4.10) and which have been neglected in (4.9). So in the following analysis, the background values of (4.9) will be assumed to be correct only up to quadratic order in the fields of (4.10). For example, the $f\overline{f}(x^-)^2$ dependence in ϕ will be neglected when considering the background value of $e^\phi F^{\alpha\beta}$ in the following analysis since this dependence only affects terms which are at least cubic order in the fields of (4.10).

It will now be shown that up to quadratic order in (4.10), $N = 2$ superconformal invariance implies that the background fields in (4.9) satisfy the type-IIB supergravity equations of motion. Note that superconformal invariance of an $N = 2$ sigma model implies up to a four-loop anomaly [14] that the Kahler metric is Ricci-flat [12], i.e.

$$\partial_M \overline{\partial}_{\overline{N}} [\log sdet(\partial\overline{\partial} K)] = 0\,, \tag{4.11}$$

where ∂ denotes derivatives with respect to $(X^l, \Theta_L, \Theta_R)$ chiral superfields and $\overline{\partial}$ denotes derivatives with respect to $(\overline{X}^{\overline{l}}, \overline{\Theta}_L, \overline{\Theta}_R)$ antichiral superfields. Defining K to be the Kahler

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potential of (4.2) where $\widehat{k} = k - X^l\overline{X}^{\overline{l}}$ and $\widehat{b} = b - 1/\sqrt{f\overline{f}}$, one finds that

$$\begin{aligned} sdet(\partial\overline{\partial}K) \, = \, f\overline{f} \, \det g \, \Bigg[& 1 - 2\sqrt{f\overline{f}}\widehat{b} + \Theta_L\Theta_R g^{l\overline{m}}\partial_l\overline{\partial}_{\overline{m}}a + \overline{\Theta}_R\overline{\Theta}_L g^{l\overline{m}}\partial_l\overline{\partial}_{\overline{m}}\overline{a} + \\ & + \left(\sqrt{\frac{f}{\overline{f}}}\Theta_L\overline{\Theta}_R + \sqrt{\frac{\overline{f}}{f}}\Theta_R\overline{\Theta}_L\right)\left(g^{l\overline{m}}\partial_l\overline{\partial}_{\overline{m}}\widehat{b} + \sqrt{f\overline{f}}d\right) + \\ & + \Theta_L\overline{\Theta}_L\Theta_R\overline{\Theta}_R(g^{l\overline{m}}\partial_l\overline{\partial}_{\overline{m}}d - (\partial_l\overline{\partial}_{\overline{m}}a)(\overline{\partial}_{\overline{l}}\partial_m\overline{a}) + (\partial_l\overline{\partial}_{\overline{l}}a)(\partial_m\overline{\partial}_{\overline{m}}\overline{a}) + \\ & + (\partial_l\overline{\partial}_{\overline{m}}\widehat{b})(\overline{\partial}_{\overline{l}}\partial_m\widehat{b}) - (\partial_l\overline{\partial}_{\overline{l}}\widehat{b})(\partial_m\overline{\partial}_{\overline{m}}\widehat{b})) + \cdots \Bigg] , \end{aligned} \tag{4.12}$$

where $g^{l\overline{m}}\partial_l\partial_{\overline{m}} = \partial_l\partial_{\overline{l}} - (\partial_m\overline{\partial}_{\overline{l}}\widehat{k})\partial_l\overline{\partial}_{\overline{m}}$ and $\cdots$ is at least cubic order in the fields of (4.10). So up to terms quadratic order in these fields, (4.11) implies the equations

$$\begin{aligned} \partial_m\overline{\partial}_{\overline{n}}(\log\det\ g - 2\sqrt{f\overline{f}}\widehat{b}) \, &= \, 0 \, , \\ \overline{\partial}_{\overline{n}}(g^{l\overline{m}}\partial_l\overline{\partial}_{\overline{m}}a) \, &= \, 0 \, , \\ \partial_n(g^{l\overline{m}}\partial_l\overline{\partial}_{\overline{m}}\overline{a}) \, &= \, 0 \, , \\ g^{l\overline{m}}\partial_l\overline{\partial}_{\overline{m}}\widehat{b} + \sqrt{f\overline{f}}d \, &= \, 0 \, , \\ g^{l\overline{m}}\partial_l\overline{\partial}_{\overline{m}}d - (\partial_l\overline{\partial}_{\overline{m}}a)(\overline{\partial}_{\overline{l}}\partial_m\overline{a}) + (\partial_l\overline{\partial}_{\overline{m}}\widehat{b})(\overline{\partial}_{\overline{l}}\partial_m\widehat{b}) \, &= \, 0 \, . \end{aligned} \tag{4.13}$$

It will now be shown that (4.13) implies that the background fields of (4.9) satisfy the type-IIB supergravity equations in string gauge. When the NS-NS $B_{\mu\nu}$ field vanishes, the type-IIB supergravity equations can be written in terms of the bispinor $F^{\alpha\beta}$ of (3.3) as

$$e^{-2\phi}(R_{\mu\nu} - 2\nabla_\mu\nabla_\nu\phi) \, = \, -\frac{1}{64}\gamma_\mu^{\alpha\beta}\gamma_\nu^{\gamma\delta}F_{\alpha\gamma}F_{\beta\delta} + \frac{g_{\mu\nu}}{64}F^{\alpha\beta}F_{\alpha\beta} \, , \tag{4.14}$$

$$(\gamma_{\mu\nu})^\alpha{}_\gamma F^{\gamma\beta}F_{\alpha\beta} \, = \, 0 \, , \tag{4.15}$$

$$R \, = \, -4\nabla_\mu\phi\nabla^\mu\phi + 4\nabla^\mu\nabla_\mu\phi \, , \tag{4.16}$$

$$\gamma^\mu_{\alpha\beta}\nabla_\mu F^{\beta\gamma} \, = \, \gamma^\mu_{\alpha\beta}\nabla_\mu F^{\gamma\beta} = 0 \, . \tag{4.17}$$

Note that (4.15) comes from varying $B^{\mu\nu}$ in the supergravity action and then setting $B^{\mu\nu} = 0$.

To verify that these supergravity equations are implied by (4.13), first note that the matrices M_α^β and $\overline{M}_\alpha^\beta$ of (3.5) satisfy

$$(\gamma^m)^{\alpha\beta}M_\beta^\gamma = (\gamma^{\overline{m}})^{\alpha\beta}\overline{M}_\beta^\gamma = M_\alpha^\beta(\gamma^{\overline{m}})_{\beta\gamma}M_\delta^\gamma = \overline{M}_\alpha^\beta(\gamma^m)_{\beta\gamma}\overline{M}_\delta^\gamma = 0 \, , \tag{4.18}$$

which implies that the background value of (4.9) for $F^{\alpha\beta}$ satisfies

$$F^{\alpha\beta}F_{\alpha\gamma} = F^{\alpha\beta}F_{\gamma\beta} = 0 \, , \tag{4.19}$$

and that the only non-zero components of $e^{2\phi}\gamma_\mu^{\alpha\gamma}\gamma_\nu^{\beta\delta}F_{\alpha\beta}F_{\gamma\delta}$ are

$$\begin{aligned} e^{2\phi}\gamma_+^{\alpha\gamma}\gamma_+^{\beta\delta}F_{\alpha\beta}F_{\gamma\delta} \, &= \, 64(\partial_l\partial_m\overline{a})(\overline{\partial}_{\overline{l}}\overline{\partial}_{\overline{m}}a) + 64(\partial_l\overline{\partial}_{\overline{m}}\widehat{b})(\partial_m\overline{\partial}_{\overline{l}}\widehat{b}) \, , \\ e^{2\phi}\gamma_-^{\alpha\gamma}\gamma_-^{\beta\delta}F_{\alpha\beta}F_{\gamma\delta} \, &= \, 256f\overline{f} \, , \\ e^{2\phi}\gamma_l^{\alpha\gamma}\gamma_{\overline{m}}^{\beta\delta}F_{\alpha\beta}F_{\gamma\delta} \, &= \, 128\sqrt{f\overline{f}}\partial_l\overline{\partial}_{\overline{m}}\widehat{b} \, . \end{aligned} \tag{4.20}$$

Also, the background value for $g_{\mu\nu}$ of (4.9) implies that the Ricci tensor $R_{\mu\nu}$ satisfies

$$\begin{aligned} R_{++} &= -g^{l\overline{m}}\partial_l\overline{\partial}_{\overline{m}}g_{++} = g^{l\overline{m}}\partial_l\overline{\partial}_{\overline{m}}(d - (\overline{\partial}_{\overline{n}}a)(\partial_n\overline{a}))\,, \\ R_{l\overline{m}} &= -\partial_l\overline{\partial}_{\overline{m}}(\log\det\ g)\,, \qquad R_{--} = R_{lm} = R_{\overline{l}\overline{m}} = 0\,. \end{aligned} \tag{4.21}$$

Putting together (4.21) and (4.13), one finds that

$$R_{++} = -(\partial_l\overline{\partial}_{\overline{m}}\widehat{b})(\overline{\partial}_{\overline{l}}\partial_m\widehat{b}) - (\partial_l\partial_m\overline{a})(\overline{\partial}_{\overline{l}}\overline{\partial}_{\overline{m}}a) \qquad \text{and} \qquad R_{l\overline{m}} = -2\sqrt{f\overline{f}}\partial_l\overline{\partial}_{\overline{m}}\widehat{b}\,. \tag{4.22}$$

So (4.14) is satisfied using the values of (4.20) and (3.14) for $(FF)_{\mu\nu}$ and ϕ. Furthermore, (4.15) is implied by (4.19), and (4.16) is implied (up to quadratic order in (4.10)) by (4.22) and the fourth equation of (4.13).

Finally, (4.17) can be verified using the background value of (4.9) for $F^{\alpha\beta}$ together with the identities $\gamma^{lm}\nabla_l\nabla_m = \gamma^{\overline{l}\overline{m}}\overline{\nabla}_{\overline{l}}\overline{\nabla}_{\overline{m}} = 0$, $\{\gamma^l,\gamma^{\overline{m}}\} = 2g^{l\overline{m}}$, and (4.18) to obtain

$$\begin{aligned} \gamma^{\mu}_{\alpha\beta}\nabla_\mu F^{\beta\gamma} &= (\gamma^l_{\alpha\beta}\nabla_l + \gamma^{\overline{l}}_{\alpha\beta}\overline{\nabla}_{\overline{l}} + \gamma^{-}_{\alpha\beta}\nabla_- + \gamma^{+}_{\alpha\beta}\nabla_+)F^{\beta\gamma} \\ &= M^{\delta}_{\alpha}\gamma^{+}_{\delta\kappa}(\gamma^{\overline{m}})^{\kappa\gamma}\nabla_l\overline{\nabla}_{\overline{l}}\overline{\nabla}_{\overline{m}}a + \overline{M}^{\delta}_{\alpha}\gamma^{+}_{\delta\kappa}(\gamma^{m})^{\kappa\gamma}\overline{\nabla}_{\overline{l}}\nabla_l\nabla_m\overline{a} + \\ &\quad + \frac{1}{4}M^{\delta}_{\alpha}\gamma^{+}_{\delta\kappa}\overline{M}_{\kappa\rho}(\gamma^m)^{\rho\gamma}\frac{\sqrt{f}}{\sqrt{\overline{f}}}\nabla_l\overline{\nabla}_{\overline{l}}\nabla_m\widehat{b} + \frac{1}{4}\overline{M}^{\delta}_{\alpha}\gamma^{+}_{\delta\kappa}M_{\kappa\rho}(\gamma^{\overline{m}})^{\rho\gamma}\frac{\sqrt{\overline{f}}}{\sqrt{f}}\overline{\nabla}_{\overline{l}}\nabla_l\overline{\nabla}_{\overline{m}}\widehat{b} + \\ &\quad + \frac{1}{4}\gamma^{+}_{\alpha\beta}w_+{}^{cd}[(\gamma_{cd})^{\beta}{}_{\delta}F^{\delta\gamma} + (\gamma_{cd})^{\gamma}{}_{\delta}F^{\beta\delta}] \end{aligned} \tag{4.23}$$

where

$$\begin{aligned} \frac{1}{4}w_+{}^{cd}(\gamma_{cd})^{\beta}{}_{\delta} &= \frac{1}{4}[\partial_l g_{++}(\gamma^+\gamma^l)^{\beta}{}_{\delta} + \overline{\partial}_{\overline{l}}g_{++}(\gamma^+\gamma^{\overline{l}})^{\beta}{}_{\delta}] \\ &= -\frac{1}{4}[\partial_l d(\gamma^+\gamma^l)^{\beta}{}_{\delta} + \overline{\partial}_{\overline{l}}d(\gamma^+\gamma^{\overline{l}})^{\beta}{}_{\delta}] \end{aligned} \tag{4.24}$$

is the + component of the spin connection to linearized order in (4.10). Using (4.13) and

$$\begin{aligned} \gamma^{+}_{\alpha\beta}(\gamma^+\gamma^l)^{\gamma}{}_{\delta}F^{\beta\delta} &= -M^{\sigma}_{\alpha}\gamma^{+}_{\sigma\kappa}\overline{M}^{\kappa}_{\rho}(\gamma^l)^{\rho\gamma}f\,, \\ \gamma^{+}_{\alpha\beta}(\gamma^+\gamma^{\overline{l}})^{\gamma}{}_{\delta}F^{\beta\delta} &= -\overline{M}^{\sigma}_{\alpha}\gamma^{+}_{\sigma\kappa}M^{\kappa}_{\rho}(\gamma^{\overline{l}})^{\rho\gamma}\overline{f}\,, \end{aligned} \tag{4.25}$$

one learns that $\gamma^{\mu}_{\alpha\beta}\nabla_\mu F^{\beta\gamma} = 0$. Similarly, one can show that $\gamma^{\mu}_{\alpha\beta}\nabla_\mu F^{\gamma\beta} = 0$.

So it has been verified up to quadratic order in the fields of (4.10) that $N = 2$ superconformal invariance of (4.2) implies the type-IIB supergravity equations of motion for the background. It would be useful to determine the complete backreaction of the background values in (4.9) and verify this to all orders in the fields of (4.10). It would also be interesting to consider more general actions than (4.2) and determine what is the most general type-IIB supergravity background that can be described as an $N = 2$ sigma model using the U(4) hybrid formalism.

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